**EE 488 Power Systems Analysis I**

**Problem Set #2:** Transformers and transmission lines **– solutions**

1. (20 pts) A single-phase, 60 Hz step-down transformer is rated at 480/240 V and 10 kVA. The secondary is connected to a 220V load that absorbs 8 kVA at 0.8 pf lagging. This is a non-ideal transformer and the series equivalent impedance of the transformer has already been calculated at 0.6 + j1.8 ohms referred to the primary side. You may neglect the transformer excitation current.
   1. (2 pts) Draw a simple transformer diagram (choose the appropriate schematic from Fig. 3.6 in the textbook) and label the diagram appropriately. Show the load connected as a box with the power information inside.

Here is the diagram, based on Fig. 3.6 (b) from the textbook:

A diagram of a circuit

Description automatically generated

* 1. (4 pts) Assume that the load voltage is 220 ∠ 0° V. What is the load current in polar form? What is the primary current in the ideal transformer part of your sketch?
  2. (4 pts) What is the transformer excitation voltage, V1, in rectangular form, needed to maintain the load at 220V, given the information found in part b.?
  3. (4 pts) Calculate the real and reactive power provided at the voltage source

Power check:

* 1. (4 pts) Calculate the load impedance referred to the primary in rectangular form

Check:

* 1. (2 pts) Calculate the real power efficiency of the transformer

1. (10 pts) Using the transformer ratings as base quantities, rework problem #1 using the per-unit system and then transform your answers back to regular units as a check on your solutions to #1.b. through #1.e. Be sure to draw any suitable diagram needed.

**(6 pts)** Here is a diagram of what we have so far:

0.026 + j0.078

**I**

0.0365 + j0.1042

**+**

0.9167

**-**

**+**

**V1**

**-**

0.84 + j0.63

0.9 + j0.793

**(4 pts)** Now to check all the answers:

The main advantage of this is that the transformer is replaced by a single impedance.

1. (15 pts) A three-phase, step-up, generation transformer is rated at 900 kVA. Each phase of the transformer is rated at 8.314kV(∆)/96kV(Y). The balanced load connected to the high voltage side operates at 50kV, 200 kVA (3Φ), @ pf = 0.7071 lagging. The primary side leakage inductive reactance is 0.1 per unit. Also:

* Winding resistance is negligible;
* Secondary leakage reactance is already referred to the primary side;
* Excitation current is neglected.
  1. (5 pts) Using 900 kVA (3-phase rating) as a power base and 96 kV as the secondary side voltage base, find all of the appropriate per-unit quantities and draw a single-phase impedance diagram. Remember that the low voltage side is delta connected.

This is an exercise in using a non-conventional voltage base.

Using Figure 3.18(b) as a reference:

**Iload = 0.739 ∠ -45°**

**Ia**

A green and blue lines

Description automatically generated

**+**

**Vload = 0.3 ∠0°**

**-**

**Ean**

**Van**

**1 : ej30°**

**j0.1**

* 1. (10 pts) Using the per-unit quantities, calculate the per-unit magnitude of the generator voltage (line-to-line) needed to sustain the load at its operating point, then calculate the actual magnitude of the generator voltage.

Using rule #2 on p. 121 of the textbook:

Again, it is useful as a check to work this problem without using the per unit system:

Using the line-to-neutral windings ratio of the step-up transformer:

Now we need to convert the transformer reactance to Ohms. We will assume that the manufacturer used the same voltage base and power base we used previously:

This is approximately the same as we got before

1. (20 pts) A single-phase generator, called G, is connected to a bus feeding a step-up transformer T1, transmission line, step-down transformer T2, and load L. A simple block diagram is shown in Fig. 1.

**Z**

**L**

**T2**

**G**

**T1**

Fig. 1. General block diagram showing interconnectivity

Table 1 contains the nameplate ratings of each component, along with manufacturer-computed per unit impedance values for G, T1, and T2. The per unit impedance values shown in Table 1 for G, T1, and T2 are based on the ratings shown in the table. The transmission line has a series reactance of Z = j6 Ω (not p.u.) with negligible series resistance. Note that all other resistances are negligible, except the load.

Under current operating conditions, the load is drawing 45 MVA at a power factor of 0.7 lagging and maintains a load voltage of 10 kV ∠ 0°.

Table 1. Ratings and impedance information for problem 4.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  |  | **Nameplate Impedance** | |
| **Component** | **MVA Rating** | **Voltage Rating (kV)** | **Resistive (p.u.)** | **Inductive (p.u.)** |
| G | 100 | 20 | 0 | 0.0125 |
| T1 | 80 | 20/400 | 0 | 0.03 |
| T2 | 80 | 300/15 | 0 | 0.0711 |
| L | 60 | 25 | TBD | TBD |

Using a system power base of 100 MVA and using voltage bases at the primary of T1 and the secondary of T2 of 10 kV, you are going to use the per unit system to analyze this simple circuit. Do the following:

* 1. (3 pts) Draw a one-line diagram of the system. Be sure to number the buses. Label the load with its calculated P and Q values.

A black and white image of a number

Description automatically generated with medium confidence

Zone 1 Zone 2 Zone 3

* 1. (6 pts) Calculate all p.u. quantities and redraw the system as an impedance diagram. Be sure to show and label (with per unit) all impedances individually, including generator, transformers, transmission line, and load. Remember that there may be several nameplate conversions necessary.

There are three zones to consider.

**Zone 1 (includes generator impedance):**

**Zone 2 (includes transformers and line impedances):**

**Zone 3 (includes load):**

j0.05

j0.2

j0.015

**I**

j0.15

A white background with green and purple lines

Description automatically generated

Vs

1.554

j1.585

* 1. (4 pts) Calculate the per unit current flowing in the system. Then find an appropriate current base and calculate the actual load current. You may assume zero phase for the load voltage.
  2. (3 pts) Find the per unit source voltage necessary to maintain the load at 10 kV.
  3. (4 pts) Recommend a suitable MVA rating for the transmission line, such that the present load demand has the capability to be increased by up to 25% (same power factor) in the future. Does this increase exceed the MVA rating of any other component? Construct a suitable table to show your results.

Here is an analysis of the other components:

**( 2 pts)** Based on the following table, the ratings on all components look fine.

|  |  |  |
| --- | --- | --- |
|  | **Old Rating (MVA)** | **Min Rating (MVA)** |
| **Generator** | 100 | 64.3 |
| **Transformer T1** | 80 | 63.2 |
| **Transmission Line** | -- | 70 |
| **Transformer T2** | 80 | 60 |
| **Load** | 60 | 56.25 |

1. (10 pts) You are analyzing an ACSR three-phase, 60 Hz, 345 kV overhead line using the characteristics of “Mallard” in Table A.4 of the text. Each phase is bundled with 2 conductors. Using basic principles (formulas – for example, see lesson reading in Canvas), and the following assumptions, find the approximate resistance of each line in mΩ/km. Note that for Mallard, there are 30 strands per conductor. Assume:
   1. Air temperature is 50° C
   2. Resistivity of aluminum of 2.7 x 10-8 Ω-m at 20° C
   3. Spiraling of the conductor – add about 2% to the overall length
   4. Add an additional 5% of resistance due to the skin effect at 60Hz

Once you have calculated the resistance for each line the “long way”, go to Table A.4 and use the appropriate column under “ra Resistance (Ohms per Conductor per Mile)” to look up the value for Mallard. Do a quick calculation using the conversion from miles to km and the fact that you have 2 conductors per phase to arrive at a line resistance value. It should be very close to your previous calculation (within about + 2 mΩ/km).

We can now find the resistance of 2 lines at 1000 m:

To correct for spiraling and skin effect:

The far-right column on the table shows Mallard with 0.1288 ohms/conductor/mile. This can be quickly converted assuming 2 conductors as follows:

1. (10 pts) Using basic principles found in the textbook and class notes, find the loop inductance and the total inductive reactance of a single-phase line that is 20 miles long. The following information is given:
   * Each of the two lines is the same, of type “Finch” from Table A.4.
   * Line spacing is 2.5 meters
   * Operating frequency is 50 Hz

The outside diameter of Finch is given as 1.293 inches. This is equivalent to 1.293/12 = 0.108 feet. We divide this by 2 to get a radius for the conductor of 0.054 ft. The line spacing is 2.5 meters, which is 8.2 ft. The calculation for inductance becomes:

Note that you could also use GMR right from the table. For Finch, this is 0.0435. This is close to the denominator value above of (0.7788)(0.054) = 0.042. It will produce essentially the same answer.

1. (15 pts) A bundled, fully transposed, three-phase line is constructed using the geometry shown below. Each phase is a bundle of three conductors code named “Mallard” in Table A.4. Spacing within each equilateral triangle bundle and between phases is shown.

A diagram of a line with dots and arrows

Description automatically generated

(10 pts) Find the total impedance in ohms per km per phase (resistance and inductive reactance) if the system is operating at 60 Hz and 50°C. You do not have to find resistance the “long way” as you did in problem 5.

Note that the spacing on the side of the triangular bundles is intended to be 0.4m. I used a text box to cover up the original value, which was 0.5m. If that text box slipped, and you used 0.5m that’s fine. Also, the spacing between bundles is supposed to be 6m. If those text boxes slipped in your version and you used 4m, that’s fine as well.

We start with the geometric mean radius (GMR) calculation for the bundle:

Then we can find the inductance for one meter:

Finally, we extend this to 1 km and use 60 Hz to find the reactance:

For resistance, we use the 50°C/60 Hz column and convert to km, with 3 in a bundle:

(5 pts) Bundling of individual phases is quite common. What are the advantages and disadvantages of using bundling in a 3-phase, aluminum transmission line?

1. Bundling tends to lower the voltage gradient between conductors in order to lessen the possibility of “corona discharge.” This occurs when the atmospheric medium between conductors becomes statically charged to the point where the medium becomes a better conductor than insulator. You have probably all seen this kind of discharge between two aluminum balls in a Van De Graaf generator.
2. Interestingly, bundling increases capacitance of the line. Normally we might think this is a bad thing, but it serves to counteract some of the line inductance. Thus, overall power factor is improved.
3. Bundling also lowers inductance (and thus reactance) per phase. Again, this improves the loss characteristic of the lines. Note that in the calculations, GMR3 is used and is a larger number than just GMR of a single conductor. Since it is in the denominator, a larger number is better.

A good university-based source on bundling is the presentation found at: <https://web.ecs.baylor.edu/faculty/lee/ELC4340/Lecture%20note/ECL%2043402022_Lect9.pdf>.